*International Journal of Theoretical Physics, Vol. 45, No. 12, December 2006 (© 2006)*
DOI: 10.1007/s10773-006-9195-z

# Competition Among Three Coupled Bose-Einstein Condensates due to Potential Deviation

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*Received February 6, 2006; accepted April 4, 2006*
*Published Online: June 9, 2006*

Three coupled Bose-Einstein condensates are investigated by the variational approach in finite potentials with potential deviation, and the effects of the potential deviation on dynamics of the three Bose-Einstein condensates are studied. The potential deviation leads to the shift of the stationary state, resets the stability condition, causes the population imbalance competition, and changes the switching and self-trapping effects on the Bose-Einstein condensates. The effect mechanism is demonstrated by performing a coordinate of classical particles moving in an effective potential field. The critical behaviors are analyzed, and are confirmed by evolution of the atom population imbalance ratio.

**KEY WORDS:** Coupled Bose-Einstein condensate; potential deviation; imbalance competition; stationary state.

**PACS:** 03.75.Fi, 32.80.Pj

## 1. INTRODUCTION

The development of Bose–Einstein condensation is nowadays responsible for many current perspectives on the potential applications, and these types of experimental apparatus offer a unique possibility for studying dynamical regimes at the frontier between the quantum and classical scenarios, for example, coherent atomic lasers (Andrews *et al*., 1997), and the new chemistry of atomic-molecular condensates (Donley *et al*., 2002). Subsequently, Bose–Einstein condensates are seen as one of the main tools to investigate, verify and improve understanding of many physical concepts and principles, for example, a long-term perspective of such coherent matter-wave devices is quantum information processing on the nanometer scale (Madison *et al*., 2000)

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Solitons are a central paradigm of nonlinear physics, and have become the basis of transoceanic communication systems in fiber optics. A mean–field description for the macroscopic BEC wave-function is constructed using the Hartree–Fock approximation and results in the Gross-Pitaevskii equation (GPE). It is well known that the GPE support solitonic solutions (Fetter and Svidzinsky, 2001). The dynamics of BECs in the presence of trap potential have recently become a subject of growing interest, and have been studied in the framework of almost all nonlinear evolution equations possessing soliton solutions (Aubry *et al.*, 1996; Raghavan and Agrawal 2000; Li *et al.* 2005, 2006; Raghavan *et al.*, 1999).

The two-well model has been used to represent two coupled BECs in a symmetric double-well potential, where the initial state with the atomic population self-trapped in one well is shown to evolve in delocalized oscillations involving both the wells. The dynamics of the asymmetric two-well model have been faced within the mean-field formulation relatively to the $\pi$-phase oscillations as well as the self-trapping effect (Smerzi *et al.*, 1997). The dynamics of three coupled BECs are investigated within a semi-classical scenario based on the standard boson coherent states and show how a approach entails a simple formulation of the dimeric regime therein studied. This allows to recognize the parameters that govern the bifurcation mechanism causing self-trapping, and paves the way to the construction of analytic solutions. The results of a numerical simulation show the three-well dynamics has a cahotic behavior (Penna and Franzosi, 2002).

But with the classic theory (variational approach) of many-body BEC model (three-well model), the switching and self-trapping effects on the three coupled BECs in trap potentials and the dynamic mechanism have not been studied in detail yet. Recently, a relevant interesting issue is to learn how to control the motion of different types of condensates, including the coupled BECs. The question then arises as to how one could affect or even guide their motion. The effect of asymmetry of the laser intensity in turn involves deviation of amplitude (strength) of the trapping potentials (Garnier *et al.*, 2004). The control of the motion of the coupled condensate solitons results in a train of self-coherent solitonic pulses. Theoretical and numerical evidence suggests that such a pulsed atomic soliton laser can be made in present experiments. One of the most important aspects of the pulsed atomic lasers is that each BEC may be unstable due to the coupling interaction among BECs. The interaction can be guided when the effects of external factors (i.e. the potential deviation) are used.

In this paper, the population imbalance competition among the three coupled BECs under deviation of the amplitude (strength) of the trap potential is investigated, and the effects of the deviation on the dynamics of the three coupled BECs are studied by means of the variational approach. Some novel results are obtained, and they remind that the effects can be used to guide the motion of the three coupled BECs.

## 2. THE THEORETICAL FORMULA

Three coupled BECs in three traps can be described by the following three coupled nonlinear Schrödinger equations (Li *et al.* 2005, 2006; Raghavan *et al.*, 1999; Smerzi *et al.*, 1997)

$$\begin{aligned}
&j\frac{\partial u_1}{\partial t}+\frac{1}{2}\frac{\partial^2 u_1}{\partial z^2}+|u_1|^2 u_1=V_1(z)\,u_1+Ku_2,\\
&j\frac{\partial u_2}{\partial t}+\frac{1}{2}\frac{\partial^2 u_2}{\partial z^2}+|u_2|^2 u_2=V_2(z)u_2+K\,(u_1+u_3)\,,\\
&j\frac{\partial u_3}{\partial t}+\frac{1}{2}\frac{\partial^2 u_3}{\partial z^2}+|u_3|^2 u_3=V_3(z)\,u_3+Ku_2,
\end{aligned}\tag{1}$$

where $u_i(i = 1, 2, 3)$ is the condensate wave-function, $K$ is the linear coupling coefficient arising out of overlaps of the transverse parts of the wave-functions, and $V_i(i = 1, 2, 3)$ is the normalized confining trap potential in the longitudinal direction ($z$ direction).

The spatial dependence is weak compared with the temporal dependence, and the three traps behave independently with the well-known ground state solution in the form of Gaussian-shape (quasi-soliton). Then we use trial wave functions below as the solutions of Eq. (1)

$$\begin{aligned}
u_1(z,t)&=\frac{N\cos^4\theta(t)}{\sqrt[4]{\pi}}\exp\left[-\frac{N^2\cos^8\theta(t)}{2}z^2+j\phi(t)+j\varphi\right],\\
u_2(z,t)&=\frac{N\sin^2 2\theta(t)}{2\sqrt[4]{\pi}}\exp\left[-\frac{N^2\sin^4 2\theta(t)}{8}z^2+j\varphi\right],\\
u_3(z,t)&=\frac{N\sin^4\theta(t)}{\sqrt[4]{\pi}}\exp\left[-\frac{N^2\sin^8\theta(t)}{2}z^2-j\phi(t)+j\varphi\right],
\end{aligned}\tag{2}$$

where $\theta(t)$, $\varphi$, $\phi(t)$ and $N_i(t) = \int_{-\infty}^{\infty}|u_i|^2 dz$ $(i = 1, 2, 3)$ are the coupling angle, the local phase, the phase difference and the number of atoms in each trap. $N = N_1 + N_2 + N_3$ is the total number of atoms in the three traps (a conserved quantity). The wave-function $u_i(i = 1, 2, 3)$ retains the Gaussian shape given by Eq. (2) in the evolution of the three BECs, but the coupling angle, the phase difference and the number of atoms in each trap become functions of time.

It is convenient to decompose the external potential $V_i (i = 1, 2, 3)$ as follows

$$\begin{aligned} V_1(z) &= U(z)[1+\sigma] \\ V_2(z) &= U(z), \\ V_3(z) &= U(z)[1-\sigma] \end{aligned} \tag{3}$$

where the additional potential $\sigma$ accounts for the time-independent potential deviation, which is assumed to be small and may take either positive or negative values. $U(z)$ is the conventional time-independent trapping potential, which is assumed to be smooth and slowly varying.

Three coupled BECs in finite traps are considered, and the trap potential is (Carr *et al.*, 2001)

$$U(z) = \begin{cases} 0 & |z| \le 1 \\ V_0 & |z| > 1 \end{cases}, \tag{4}$$

where $U(z)$ represents the square well potential, and $V_0$ is the amplitude of the potential. This potential gives analytic solutions, unlike harmonic traps, which in one dimension do not give rise to analytic solutions. It has the advantage of having a direct analog in the linear Schrödinger equation, for which the stationary states have been worked out completely.

The averaged Lagrangian of Eq. (1) can be defined as usual variational approach

$$\begin{aligned} L(t) = \int_{-\infty}^{\infty} &\left\{ \sum_{i=1}^{3} \left[ \frac{j}{2}\left(u_i * \frac{\partial u_i}{\partial t} - u_i \frac{\partial u_i *}{\partial t}\right) - \frac{1}{2}\left|\frac{\partial u_i}{\partial z}\right|^2 + \frac{1}{2}|u_i|^4 - V_i |u_i|^2 \right] \right. \\ &\left. - K(u_1 u_2 * + u_3 u_2 * + u_2 u_1 * + u_2 u_3 *) \right\} dz \\ = -N \cos 2\theta \frac{d\phi}{dt} &- aN^3\left(1 - \frac{3}{4}\sin^2 2\theta\right)^2 - \frac{3a}{32}N^3 \sin^6 2\theta \\ &- bV_0 N(1 + \sigma \cos 2\theta) - \sqrt{2}KN\left[\frac{(1+\cos 2\theta)\sin^2 2\theta}{\sqrt{5 - 6\cos 2\theta + 5\cos^2 2\theta}} \right. \\ &\left. + \frac{(1-\cos 2\theta)\sin^2 2\theta}{\sqrt{5 + 6\cos 2\theta + 5\cos^2 2\theta}}\right]\cos\phi, \end{aligned} \tag{5}$$

where $a = (\frac{1}{4} - \frac{1}{2\sqrt{2\pi}})$, and $b = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \exp[-z^2]\,\mathrm{d}z - \int_{-1}^{1} \frac{1}{\sqrt{\pi}} \exp[-z^2]\,\mathrm{d}z = 0.158$.

The equations of motions for the coupling angle and the phase difference are obtained from the averaged Lagrangian using $dL(t)/d\sigma - d[dL(t)/d\dot{\sigma}]/dt =$

$0\,(\sigma = \theta,\ \phi)$, and the following equations are obtained

$$\frac{d\theta}{dt} = \left[\frac{(1+\cos 2\theta)\sin 2\theta}{2\sqrt{5-6\cos 2\theta+5\cos^2 2\theta}} + \frac{(1-\cos 2\theta)\sin 2\theta}{2\sqrt{5+6\cos 2\theta+5\cos^2 2\theta}}\right]\sqrt{2}K\sin\phi,$$

$$\begin{aligned}\frac{d\phi}{dt} &= -\frac{3a}{8}N^2\cos 2\theta\,(5+3\cos 2\theta) + \frac{9a}{16}N^2\sin^4 2\theta\cos 2\theta - bV_0N\sigma \\ &+\sqrt{2}K\left[\frac{(1+\cos 2\theta)^3\cos 2\theta - 4\sin^2 2\theta(1-\cos 2\theta)^2}{\sqrt{(5-6\cos 2\theta+5\cos^2 2\theta)^3}}\right. \\ &\left.+\frac{(1-\cos 2\theta)^3\cos 2\theta + 4\sin^2 2\theta\,(1+\cos 2\theta)^2}{\sqrt{(5+6\cos 2\theta+5\cos^2 2\theta)^3}}\right]\cos\phi.\end{aligned} \tag{6}$$

The atom population imbalance ratio between the two side traps is given by

$$\Delta R\,(t) = R_1\,(t) - R_2\,(t) = \frac{N_1\,(t) - N_3\,(t)}{N} = \cos 2\theta\,(t)\,, \tag{7}$$

where $R_1\,(t) = \frac{N_1-N_2}{N} = 3\cos^2\theta\,\cos(\theta-\alpha)\cos(\theta+\alpha)$ and $R_2\,(t) = \frac{N_3-N_2}{N} = 3\sin^2\theta\,\sin(\theta-\alpha)\sin(\theta+\alpha)$ are the atom population transferring ratios from the both end traps to the middle trap, and $\alpha = \arg tg\sqrt{2}$.

In order to study the atom population imbalance competition among the three coupled BECs in the framework of Eq. (1), the equations of motions for the population imbalance ratio and the phase difference are transformed from the Eqs. (6) and (7) into

$$\frac{d\Delta R}{dt} = \left[\frac{(1+\Delta R)\sqrt{1-\Delta^2 R}}{2\sqrt{5-6\Delta R+5\Delta^2 R}} + \frac{(1-\Delta R)\sqrt{1-\Delta^2 R}}{2\sqrt{5+6\Delta R+5\Delta^2 R}}\right]\sqrt{2}\sin\phi,$$

$$\begin{aligned}\frac{d\phi}{dt} &= -\frac{3a\Lambda}{8}\Delta R\,(5+3\Delta R) + \frac{9a\Lambda}{16}(1-\Delta^2 R)^2\Delta R - \delta \\ &+\sqrt{2}\left[\frac{(1+\Delta R)^3\,\Delta R - 4(1-\Delta^2 R)\,(1-\Delta R)^2}{\sqrt{(5-6\Delta R+5\Delta^2 R)^3}}\right. \\ &\left.+\frac{(1-\Delta R)^3\,\Delta R + 4(1-\Delta^2 R)\,(1+\Delta R)^2}{\sqrt{(5+6\Delta R+5\Delta^2 R)^3}}\right]\cos\phi,\end{aligned} \tag{8}$$

where the time is rescaled as $Kt \to t$. The dimensionless parameter $\Lambda = N^2/K$ is relative interaction strength which represents the relative strength of nonlinear interatom interaction in each trap concerning the linear inter-trap coupling resulting from the proximity of the three traps.

The dimensionless parameter $\delta = bV_0N\sigma/K$ is the normalized potential deviation which represents the relative deviation strength among three trap potentials.

The population imbalance ratio and the phase difference are canonically conjugate. With the equations $d\Delta R/dt = \partial H/\partial\phi$ and $d\phi/dt = -\partial H/\partial\Delta R$, the Hamiltonian of the three coupled BECs is given by

$$H = a\Lambda\left[1 - \frac{3}{4}(1-\Delta^2R)\right]^2 + \frac{3a\Lambda}{32}(1-\Delta^2R)^3 + \delta\Delta R$$
$$+\sqrt{2}\left[\frac{(1+\Delta R)(1-\Delta^2R)}{\sqrt{5-6\Delta R+5\Delta^2R}} + \frac{(1-\Delta R)(1-\Delta^2R)}{\sqrt{5+6\Delta R+5\Delta^2R}}\right]\cos\phi. \quad (9)$$

Equations (8) and (9) determine the competition dynamics of the three coupled BECs.

## 3. THE SHIFT OF STATIONARY STATES

The stationary states can be obtained by setting the time derivatives in Eq. (8) to zero, and the stationary states in absence of the potential deviation are

$$\Delta R_0 = 0,$$
$$\phi_0 = 0,\ \pi; \quad (10)$$

and

$$\begin{aligned}\Delta R_0 &= 1, & \phi_0 &= \arccos(3a\Lambda/\sqrt{2});\\ \Delta R_0 &= -1, & \phi_0 &= \arccos(-3a\Lambda/4\sqrt{2}).\end{aligned} \quad (11)$$

The stability issue can be discussed by performing a standard linear stability analysis for the stationary states. Considering the stationary states (10), their stability issue can be discussed. We find that the in-phase and out-of- phase stationary states are stable with respect to small perturbations when the relative interaction strength $\Lambda$ is smaller than $96\sqrt{2}/175\sqrt{5a}$ (about 6.926).

When the potential deviation is considered, from Eq. (8) the stationary states (10) are replaced by

$$\Delta R_0 = \frac{400\sqrt{5}\delta}{288\sqrt{2} - 525\sqrt{5}a\Lambda}, \quad (12)$$

and the stability condition is reset by $[-\frac{7a\Lambda}{16} - \frac{15a\Lambda\Delta R_0}{8} + \frac{6}{25}\sqrt{\frac{2}{5}}] > 0$ for the stationary state. We can see the potential deviation leads to the shift of the stationary state, and changes the atom distribution of the stationary state in each trap. For example, the atom numbers of the stationary state (10) in three traps are $N_1 = N_3 = \frac{N}{4}$ and $N_2 = \frac{N}{2}$, but the atom numbers of the stationary state (12) are changed into (second-order reserved)

$$N_1 = \frac{N}{4} + \frac{200\sqrt{5}\delta N}{288\sqrt{2} - 525\sqrt{5}a\Lambda} + \frac{1}{4}\left(\frac{400\sqrt{5}\delta N}{288\sqrt{2} - 525\sqrt{5}a\Lambda}\right)^2,$$

$$N_2 = \frac{N}{2} - \frac{1}{2}\left(\frac{400\sqrt{5}\delta N}{288\sqrt{2} - 525\sqrt{5}a\Lambda}\right)^2, \quad (13)$$

$$N_3 = \frac{N}{4} - \frac{200\sqrt{5}\delta N}{288\sqrt{2} - 525\sqrt{5}a\Lambda} + \frac{1}{4}\left(\frac{400\sqrt{5}\delta N}{288\sqrt{2} - 525\sqrt{5}a\Lambda}\right)^2.$$

Specially, the stability condition can be controlled by changing the potential deviation. For example, the stability condition of the stationary state (12) can be realized if

$$\delta > \frac{175\sqrt{5}a\Lambda - 96\sqrt{2}}{250a\Lambda}\left(\frac{7a\Lambda}{16} - \frac{6\sqrt{2}}{25\sqrt{5}}\right) \quad (14)$$

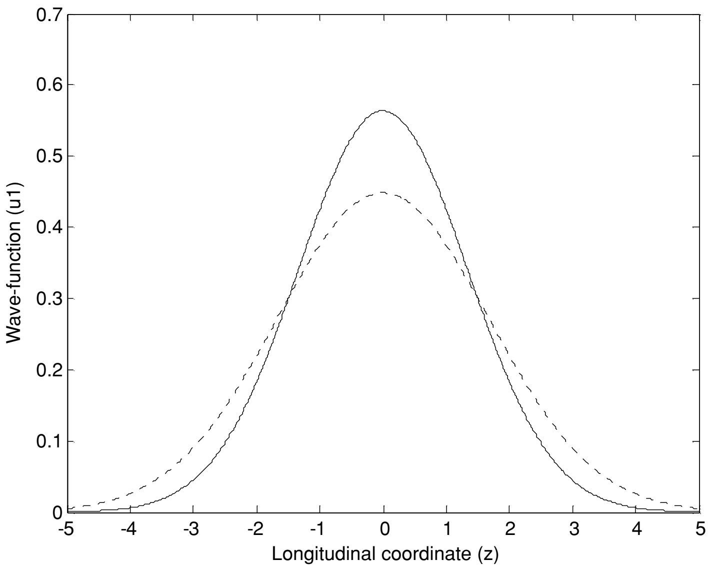


**Fig. 1.** The condensate wave-function ($u_1$) of the stable stationary state (12) versus the longitudinal coordinate ($z$). The solid: $\delta = 0$; The dotted: $\delta = 0.038$.

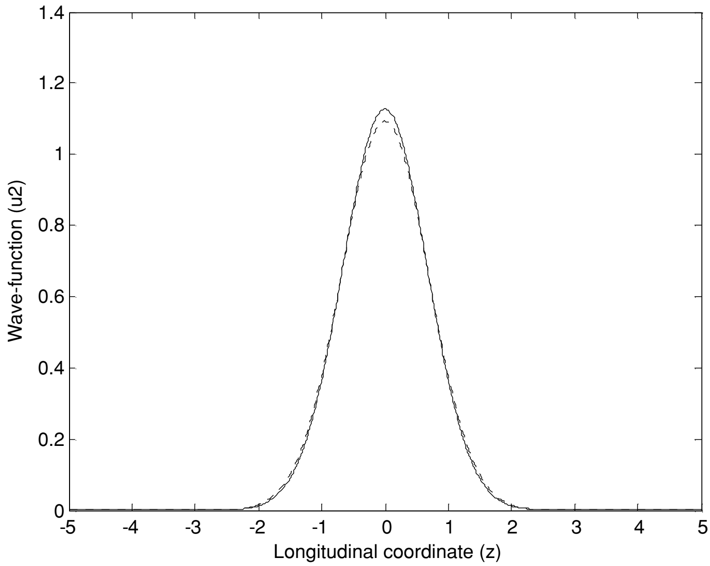


**Fig. 2.** The condensate wave-function ($u_2$) of the stable stationary state (12) versus the longitudinal coordinate ($z$). The solid: $\delta = 0$; The dotted: $\delta = 0.038$.

Figures 1–3 show the condensate wave-functions of the stable stationary state (12) versus the longitudinal coordinate ($z$ coordinate) for without and with potential deviation. The selected system parameters are that amplitude of the potential is $V_0 = 2$, the total number of atoms is $N = 3$, the coupling coefficient is $K = 1.0$, and the relative interaction strength is $\Lambda = N^2/K = 9$. The potential deviations are selected as $\sigma = 0$ and 0.04, which correspond to the normalized potential deviation $\delta = 0$, and 0.038. We can see the wave-functions can be controlled by changing the potential deviation, and there is the obvious population competition between the both end traps. These results indicate that the behaviors of the three BECs would be sensitive to the change of the potential deviation, which could be changed most easily by changing the deviations of the laser intensity which in turn involves deviations of the trapping potentials.

## 4. EFFECTIVE POTENTIAL FIELD

It is helpful for us to demonstrate the effect mechanism to analyze the dynamical atom population imbalance ratio $\Delta R(t)$ as if it were a coordinate of a classical particle ($\Delta R$ -particle) moving in an effective potential field. The phase difference $\phi(t)$ is eliminated from Eq. (9) using the conserved Hamiltonian, and

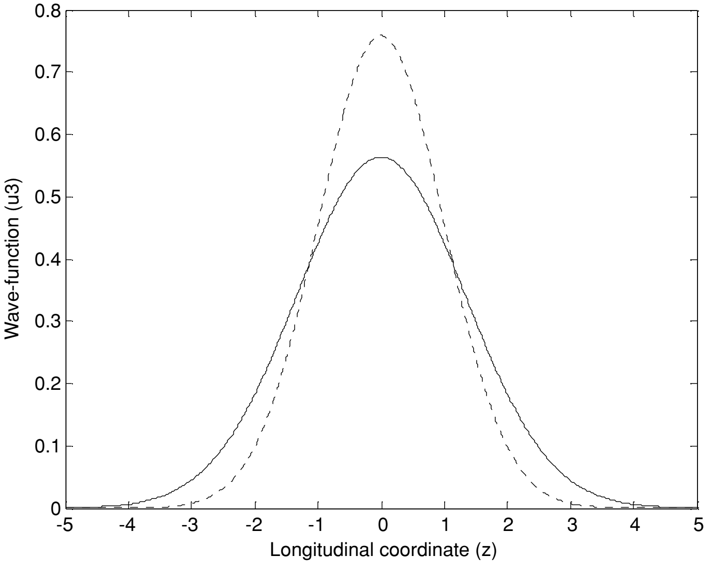


**Fig. 3.** The condensate wave-function ($u_3$) of the stable stationary state (12) versus the longitudinal coordinate ($z$). The solid: $\delta = 0$; The dotted: $\delta = 0.038$.

the following equation of motion is obtained for the coupling angle $R(t)$ alone

$$\frac{d\Delta R\,(t)}{dt} = F\,(\Delta R)\,[1 - f\,(\Delta R)]^{1/2}\,, \tag{15}$$

where the normalized function $F(\Delta R)$ and the effective potential $f(\Delta R)$ are

$$\begin{aligned} F\,(\Delta R) &= \sqrt{2}\left[\frac{(1+\Delta R)(1-\Delta^2 R)}{\sqrt{5-6\Delta R+5\Delta^2 R}} + \frac{(1-\Delta R)\,(1-\Delta^2 R)}{\sqrt{5+6\Delta R+5\Delta^2 R}}\right], \\ f\,(\Delta R) &= \left\{H\,[R\,(0)\,,\phi\,(0)] - a\Lambda\left[1-\frac{3}{4}(1-\Delta^2 R)\right]^2\right. \\ &\qquad \left. - \frac{3a\Lambda}{32}(1-\Delta^2 R)^3 - \delta\Delta R\right\}^2 \Big/ F^2\,(\Delta R)\,, \end{aligned} \tag{16}$$

where $H\,[R\,(0)\,,\ \phi\,(0)]$ is the initial Hamiltonian.

We can find that $d\Delta R/dt = 0$ gives $\Delta R = \Delta R(0)$. The energy of the $\Delta R$-particle is normalized as 1, and the effective potential field to hinder $R$-particle moving is the normalized effective potential $f\,(\Delta R)$. When the energy of $\Delta R$-particle is smaller than the effective potential $f(\Delta R)$ (namely, $f\,(\Delta R) \geq 1$), the atom population imbalance ratio $\Delta R$ is constant as if the $\Delta R$-particle is confined because of its low energy. When the energy of the $\Delta R$-particle is larger than the

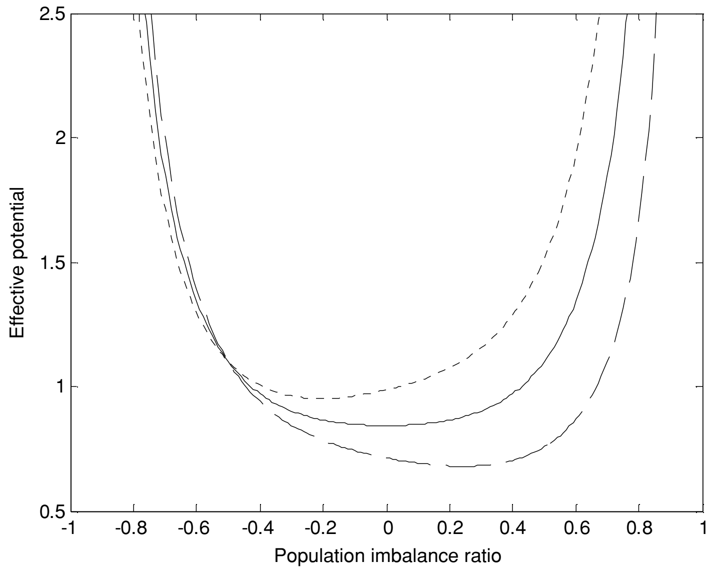


**Fig. 4.** The effective potential versus the atom population imbalance ratio with the Hamiltonian of $H[\phi(0) = 0, \Delta R(0) = -0.5]$. The dashed: $\delta = -0.038$; The solid: $\delta = 0$; The dotted: $\delta = 0.038$.

effective potential $f(\Delta R)$ (namely, $f(\Delta R) < 1$), the atom population imbalance ratio $\Delta R$ varies as if the $\Delta R$-particle has enough energy to cross over the obstruct which hinders its moving. Figure 4 show the effective potential $f(\Delta R)$ versus the atom population imbalance ratio $\Delta R$ for the different potential deviations. The selected system parameters are the same as those in Fig. 1. The potential deviation is selected as $\sigma = -0.04$, 0, and 0.04, which correspond to the normalized potential deviation $\delta = -0.038$, 0, and 0.038. The initial atom population imbalance ratio is $R(0) = -0.5$, and the initial phase difference is $\phi(0) = 0$. We can see the effective potential $f(R)$ is an approximately parabola function of one bottom. When there is no potential deviation ($\delta = 0$), the bottom is located at the atom population imbalance ratio of $\Delta R = 0$, and only the bottom is below the energy of the $\Delta R$-particle. As a result the moving of the $\Delta R$-particle is confined in the range near $\Delta R = 0$. When there is the negative potential deviation ($\delta = -0.038$), the bottom shifts along $\Delta R > 0$ and decreases, and motion of the $\Delta R$-particle is free within the large range. When there is the positive potential deviation ($\delta = -0.038$), the bottom shifts along $\Delta R < 0$ and increases, and the effective potential changes in substance compared with that with the negative potential deviation. The the $\Delta R$-particle moves within very small range near the shifted bottom. These features illuminate that the stationary states are changed by the potential deviation, the

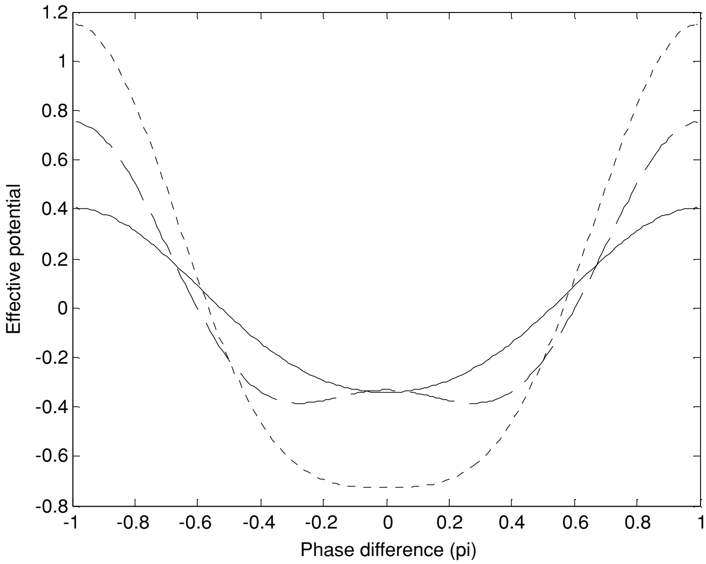


**Fig. 5.** The effective potential versus the phase difference. The dashed: $\delta = -0.038$; The solid: $\delta = 0$; The dotted: $\delta = 0.038$.

atom numbers of the stationary state are redistributed and there is the population imbalance competition among the three BECs.

Linearizing Eq. (8) in $\Delta R$ only, the Eq. (8) reduces to the very simple form

$$\frac{d^2\phi}{dt^2} = -\frac{21a\Lambda}{16}\sqrt{\frac{2}{5}}\sin\phi - \frac{9a\Lambda\Delta R_0}{8}\sqrt{\frac{2}{5}}\sin\phi - \frac{27a\Lambda\Delta^2 R_0}{8}\sqrt{\frac{2}{5}}\sin\phi + \frac{8}{125}\sin 2\phi + \frac{117\Delta^2 R_0}{125}\sin 2\phi. \quad (17)$$

This suggests a mechanical analogy in which particles of the spatial coordinate $\phi$ move in the effective potential below

$$V_{\text{eff}}(\phi) = -\frac{21a\Lambda}{16}\sqrt{\frac{2}{5}}\cos\phi - \frac{9a\Lambda\Delta R_0}{8}\sqrt{\frac{2}{5}}\cos\phi - \frac{27a\Lambda\Delta^2 R_0}{8}\sqrt{\frac{2}{5}}\cos\phi + \frac{4}{125}\cos 2\phi + \frac{117\Delta^2 R_0}{250}\cos 2\phi. \quad (18)$$

Figure 2 shows the effective potential versus the phase difference. The system parameters are selected as the same as those in Fig. 1. We see that the effective potential $V_{\text{eff}}(\phi)$ is a potential of a bottom around $\phi = 0$, where the particles

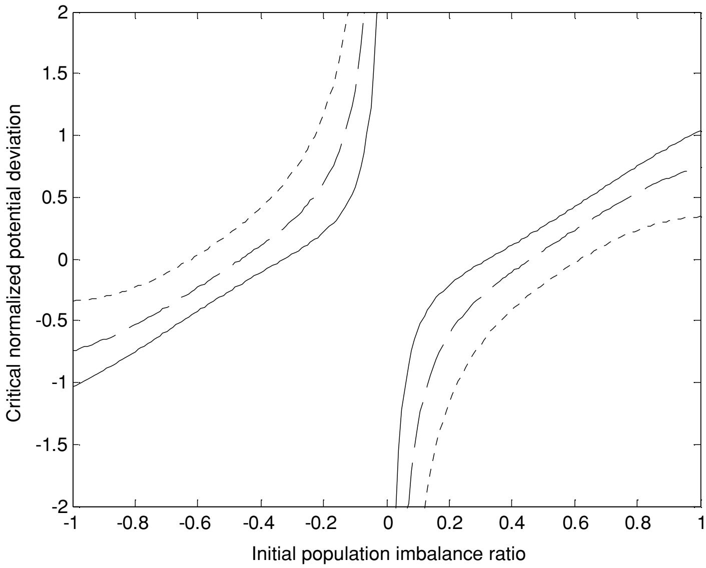


**Fig. 6.** The critical potential deviation versus the initial atom population imbalance ratio for different relative interaction strengths. The solid: $\Lambda = 4$; The dashed: $\Lambda = 9$; The dotted: $\Lambda = 16$.

can oscillate. The property of the effective potential depends strictly on the trap potential deviation. For example, there are two very shallow valleys apart from $\phi = 0$ in presence of the negative potential deviation ($\delta = -0.038$) in the effective potential, where the particles can oscillate in very large range. The two valleys disappears and the trap potential become deep in absence of the potential deviation ($\delta = 0$), where the particles can free oscillate become small. The bottom of potential becomes deep as the potential deviation becomes large. The particle oscillates within very small range as the potential deviation ($\delta = 0.038$) is large enough, and the particles may be stabilized around $\phi = 0$.

## 5. ATOM POPULATION IMBALANCE COMPETITION AMONG THE THREE COUPLED BECS

The switching and self-trapping effects on the atom population imbalance are like the pendulum bob swing in an oscillation manner. When the initial conditions are selected, there are multiple eigen-functions of the GPE, and critical behavior depends on system parameters and initial condition. The energy is $H = \frac{5a}{32}\Lambda + 2\sqrt{\frac{2}{5}}\cos\phi$ corresponding to the state of $\Delta R = 0$. The critical behavior of the three BECs can be analyzed with the energy conservation constraint, and from

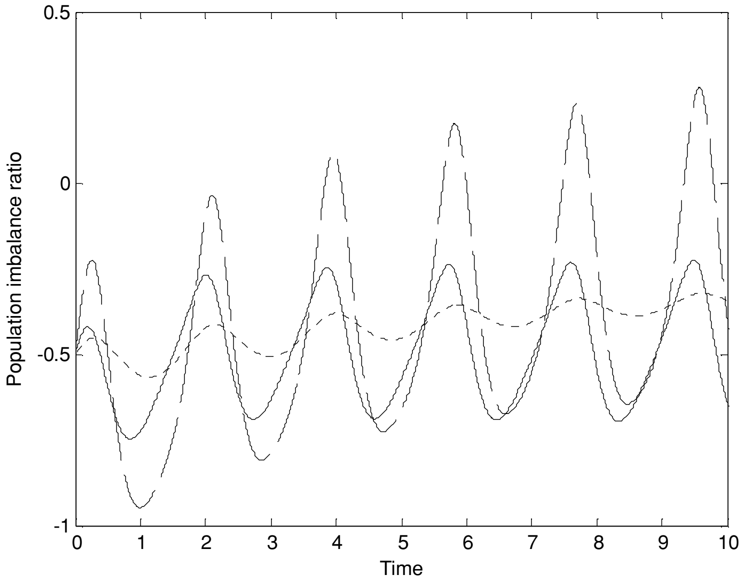


**Fig. 7.** The atom population imbalance ratio versus the time with the relative interaction strength of $\Lambda = 9$. The dashed: $\delta = -0.038$; The solid: $\delta = 0$; The dotted: $\delta = 0.038$.

Eq. (9) we can see the state of $\Delta R = 0$ are inaccessible at any time if

$$\delta > \delta_c = \left\{ \frac{2\sqrt{2}}{\sqrt{5}} - \frac{5a\Lambda}{32} - \frac{\sqrt{2}\,[1 + \Delta R\,(0)]\,[1 - \Delta^2 R\,(0)]}{\sqrt{5 - 6\Delta R\,(0) + 5\Delta R^2\,(0)}} \right. - \frac{\sqrt{2}[1 - \Delta R\,(0)][1 - \Delta^2 R\,(0)]}{\sqrt{5 + 6\Delta R\,(0) + 5\Delta R^2\,(0)}} - a\Lambda \left\{ 1 - \frac{3}{4}[1 - \Delta^2 R\,(0)] \right\}^2 \left. - \frac{3a\Lambda}{32}[1 - \Delta^2 R\,(0)]^3 \right\} / \Delta R\,(0)\,. \tag{19}$$

In Fig. 3, we plot the critical potential deviation versus the initial atom population imbalance $\Delta R(0)$ by numerically solving Eq. (19) for different relative interaction strengths. We find that the different initial conditions correspond to the different critical normalized potential deviation, and these results show that the dynamics of the three coupled BECs strictly depend on the system parameters, the potential deviation and the initial condition.

The oscillation dynamics of the atom population imbalance ratio among the three coupled BECs are shown in Fig. 4 by solving the Eqs. (7) and (8) with a fourth order variable-step Runge-Kutta algorithm for the different normalized potential deviations. The selected system parameters and the initial conditions are the same as those in Fig. 1. The normalized critical potential deviation is $\delta_c = -0.018$ by Eq. (19), and the stable condition of the stationary states (12) is $\delta > 0.016$ by Eq. (14) under the given system parameters. We can see the atom population imbalance ratios vary depending on the normalized potential deviation as time changes. For example, when there is the negative potential deviation (the corresponding normalized potential deviation of $\delta = -0.038$) in the BEC system, the population imbalance ratios vary in a relatively large and periodical fluctuation manner, and these features show that there are population imbalance competition and switching effects on the three BECs. When there is no the potential deviation (the corresponding normalized potential deviation of $\delta = 0$) in the BEC system, the atom population imbalance ratios vary in a relatively small and periodical fluctuation manner, and there is the population imbalance competition among the three BECs. The state of $\Delta R = 0$ are in accessible because the normalized critical potential deviation is smaller than zero, and the oscillation of the atom population imbalance ratio is confined within the range of $\Delta R < 0$. In other hand, the stationary state (12) is accessible but unstable with respect to small perturbations because the relative interaction strength ($\Lambda = 9$) is large than $96\sqrt{2}/175\sqrt{5}a$ (about 6.926). When there is the potential deviation whose the normalized potential deviation is larger than the critical value (corresponding normalized potential deviation of $\delta = 0.03792$), there is self-trapping effect on the three BECs. The atom population imbalance ratios damply oscillates around a fixed value with small amplitudes, and finally is located at the value such as $\Delta R = \Delta R_0 = \frac{400\sqrt{5}\delta}{288\sqrt{2}-525\sqrt{5}a\Lambda} \approx -0.278$, which is the atom population imbalance ratio at the stationary state (12). Because the potential deviation is larger than 0.0164, the stationary solutions (12) are stable with respect to small perturbations. These results demonstrate the population competition depends strictly on the system parameters, the potential deviation and the initial condition.

## 6. CONCLUSION

The competition among the three coupled BECs are investigated by the variational approach in finite potentials with potential deviation, and the effects of the potential deviation on the dynamics of the thee BECs are analyzed. The potential deviation leads to the shift of the stationary state, resets the stability condition, causes the population imbalance competition, and changes the switching and self-trapping effects on the three coupled BECs. For example, the stationary state is replaced, and the stable condition is changed due to the existence of the potential deviation. The atom number of the stationary state in each trap is redistributed, and can be controlled by changing the potential deviation. The trajectories with the energy conservation constraint in the phase space formed by the atom population imbalance ratio and the phase difference shows that there is the population imbalance competition among the three BECs, and the effect mechanism is demonstrated by performing a coordinate of classical particles moving in an effective potential field. The critical behaviors among the three BECs are analyzed with the energy conservation constraint, and are confirmed by evolution of the atom population imbalance ratio versus time. These results remind that the behaviors of the three BECs would be sensitive to the change of the potential deviation, and the effects can be used to control the motion of the three coupled BECs.

## ACKNOWLEDGMENTS

The authors undertook this work with the support of the Fund of Natural Science of Hubei Province under Grant No. 2004ABA112, and the Research Program of The Hong Kong Polytechnic University under Grant No. A-PA2Q.

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